DESIGN GUIDELINES FOR HIGH-PRESSURE PIPE FITTINGS

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ABSTRACT

There are currently no design standards which govern the dimensions of pipe fittings such as elbow thickness and tee internal radii. In an effort to determine if there is a problem and to provide guidance in evaluating fittings of unknown origin, a study was made on long radius 90 degree elbows, size-on-size tees, and reducing tees with sizes ranging from 4 to 36 inches. Finite element techniques were employed to compare the stresses in these fittings to those determined using the Stress Intensification Factors (SIF) from ANSI B31.8 and burst calculations. Experimental verification was performed via burst tests on several elbow and tee fittings.

The SIF values appear to be unconservative for elbows under external loading unless the elbows are considerably thicker than the adjoining pipe (by at least 125 percent). The SIF values for tees in B31.8 are very general in terms of dimensional scope (only employing run dimensions) and tend to be unconservative when considering bending loads. The burst tests indicate that tensile strength is the most critical material property when predicting burst pressure, and that yield strength is not significant to either burst pressure or fatigue due to cyclic loading.

The findings of this investigation indicate that users of pipe fittings should audit burst tests and the means by which fittings are qualified. Implementation of dimensional rules could also be of benefit to both the user and the industry as a whole.

INTRODUCTION

The typical maximum design pressure in gas pipelines has been approximately 70 bar (1029 psi). There is a current trend to push this operating pressure higher for economic advantages. In addition to the anticipated benefits, there are a number of technical questions which must be raised in the process of going to higher design pressures as high as 210 bar (3046 psi). Because of the higher pressures, higher strength steels, such as Y65 and X65, will likely be used; however, these steels do not tend to be as ductile and are often not as forgiving as the lower grade steels.

Another possible concern is the design standards which currently exist for pipe fittings. The current standards are ANSI B31.8 (2)*, ANSI B16.9 (3), and MSS SP-75 (4), but these only provide basic dimensional information on fittings and do not include design guidelines. These standards use hydrostatic proof testing for design verification, with one test allowed to qualify other fittings on the basis of dimensions. This can pose some significant problems when one considers that these standards allow an elbow with a D/t ratio of 24 to qualify elbows with D/t ratios from 8 to 48. This variation may be significant, especially when considering the attempts of industry to increase the design pressures within piping systems.

* - Numbers in parentheses refer to publications cited in the *Reference* section of this paper

Several specific values from the above mentioned standards have limited verification. The *i* factors and flexibility factors from ANSI B31.8 are applicable to several pipe fittings including elbows and tees. These coefficients are based on the work of Mr. A.R.C. Markl and others from Tube Turns in the 1940's and 1950's (13-15). The tests were performed primarily on 4 inch Schedule 40 pipes with standard wall thicknesses. The results of these tests were used in creating the Stress Intensification Factors for fittings such as elbows and tees. Although these tests were performed on a very specific size range, the design factors based on these results are currently applied to a wide size range of fittings. There is some question concerning the validity of this practice.

Finally, there have been a number of instances where the pedigree of a fitting (material, yield strength, ultimate strength, etc.) are not known. To verify design integrity, some means of predicting the performance of fitting of unknown origin is needed. All of these needs are the motivation for the work reported in this paper.

FINITE ELEMENT RESULTS AND COMPARISON WITH EXISTING STANDARDS

The purpose of the finite element analysis was to determine stress concentrations factors as a result of different loading conditions and compare them with existing standards. The models were created using the PATRAN modelling package and converted for analysis with ABAQUS. The models were assumed perfectly elastic with a Young's modulus of 30 million psi and a Poisson's ratio of 0.3. The elements used by ABAQUS for processing were the S4R5 type which are four-noded elements possessing five degrees of freedom (three displacements and two in-plane rotations). The following sections discuss the specific details associated with the elbow and tee finite element models.

Finite Element Analysis of Elbows

Internal pressure, in-plane and out-of-plane bending moments were applied to each elbow to study the effects of different loading conditions. **Table 1** provides the size ranges and dimensions used for the analysis. The elbow models were all built using half-symmetry and adjacent pipe extensions (five diameters in length) were added to each fitting. **Figure 1** shows the mesh arrangement used to model the elbows. The procedures used to study the response of the elbow fittings to the specified loading conditions are discussed below.

Internal Pressure Loading

A symmetry plane was imposed on the bottom surface of the model that allowed rotation only about the X-axis. Two methods were used to verify the finite element stresses. The first method employs classical equations for hoop and axial stresses. The second method uses a modified version of the hoop stress equation known as the *Lorenz Equation* (5). This equation is found in **Equation (1)** and provides the theoretical membrane hoop stress at any region within the elbow.

$$\sigma_{hoop} = \frac{Pr}{t}\left[\frac{\frac{R}{r} + \sin\frac{\phi}{2}}{\frac{R}{r} + \sin\phi}\right] \tag{1}$$

The variables *R, r, t,* and ϕ are defined in **Figure 2**. The elbows associated with this study were long radius elbows where R=3r, so that **Equation (1)** reduces to,

$$\sigma_{hoop} = 1.25\frac{Pr}{t}$$

* - Numbers in parentheses refer to publications cited in the *Reference* section of this paper

Figure 4 provides the stress concentration factors based on the hoop and peak stresses. This figure shows the hoop stress/internal pressure for plain pipes, non-reinforced elbows, elbows reinforced by 1.25 (ANR), a data point from an actual tee, and a comparison of stresses in pipes with plain dents (28). This data shows that unreinforced elbows are much higher stressed than plain pipes or reinforced elbows, but not as highly stressed as typical plain dents (at least on a local basis). The amount of material is of course much larger for an elbow than for a typical dent.

In-plane Bending

The in-plane bending case employed the same geometric arrangement as the internal pressure case with similar boundary conditions. Because of the applied moment, one end of the pipe was fixed and the moment was applied to the other end. A wagon wheel was built using beam elements which were attached to the free end of the pipe. A moment was applied to the center of the wagon wheel to transfer the moment to the rest of the piping assembly. This method is simple and minimizes stress concentrations at the end of the pipe.

The stress concentration factors which resulted from the applied in-plane moments are listed in **Table 1** along with the flexibility factors. The FEA SCF values were calculated by dividing the finite element peak stress in the elbow by the nominal axial stress in the pipe caused by the applied bending moment. The applied moment was calculated for each fitting so that a nominal stress of 20,000 psi would occur in a pipe with the same nominal thickness and diameter as the elbow. Also listed in **Table 1** are the Stress Intensification Factor, i, and the flexibility characteristic, h, from Appendix E of the ASME/ANSI B31.8 publication (2). The equations for the flexibility characteristic and in-plane stress concentration factor per ANSI B31.8 are as follows,

$$h = \frac{TR_1}{r_2^2} \tag{3}$$

where T is the thickness, R_1 the bend radius, and r_2 the nominal cross-section radius. The stress intensification factor for in-plane bending is calculated using **Equation (4).**

$$i_1 = \frac{0.9}{h^{\frac{2}{3}}} \tag{4}$$

The flexibility factors determined using finite element analyses were calculated by determining the rotation of one end of the elbow relative to the other fixed end and then dividing this relative rotation by the nominal rotation of a straight section of pipe under the same bending load. This value is found using **Equation (5)**,

$$\theta_{nom} = \frac{MR}{2Er_m^3 t} \tag{5}$$

where M is the applied moment, $R=3r_m$, and E is the modulus of elasticity. The flexibility factors from ANSI B31.8 are generally in agreement with those from the FEA work, except in the cases where complex geometries exist (such as the *ANR* reinforcement type) and the variations in thickness can not be incorporated into the general B31.8 equation.

The discrepancy which exists between the FEA values and the ANSI B31.8 i factors has been documented previously. The FEA SCF results are approximately twice what the ANSI values are calculated to be. This difference was noted by Thomas (24) as well as by Moore and Rodabaugh (25). **Figure 5** plots this in-plane SCF and shows that the actual stress intensity factor is about twice the B31.8 value for an unrenforced elbow.

Out-of-plane Bending

The final loading condition applied to the elbow models was out-of-plane bending. Once again, half-symmetry was used; however, because of this bending case the boundary conditions are different. The plane of symmetry was assumed as anti-symmetric. This is required because the elbow bends out of the Y-Z plane (refer to **Figure 1** for orientation).

The highest stresses in the model occurred in an area which was 45 degrees above the crotch toward the end with the applied moment. These results agree with the experimental findings of Markl.

The FEA SCF values are tabulated with the in-plane results in **Table 1**. As with the previous loading case, the stress concentration factors are calculated by dividing the peak stress intensity in the model by the nominal axial stress that would occur in a straight pipe under the same loading conditions. The stress intensification factor per B31.8 for out-of-plane bending is calculated using **Equation (5).**

$$i_1 = \frac{0.75}{h^{\frac{2}{3}}} \tag{6}$$

As with the in-plane results, actual SCF values are generally higher than those calculated using B31.8.

Finite Element Analysis of Tees

The information presented below discusses the finite element analyses which were preformed on the size on size and reducing tees. As with the elbow models, loadings of internal pressure, in-plane and out-of-plane bending were considered.

Internal Pressure Loading

The internal pressure case utilized quarter-symmetry with approximately 900 elements. The model was constrained using the ABAQUS symmetry commands on nodal sets which lay on the planes of symmetry. Loading was accomplished by applying an internal pressure of 1,000 psi to the inside face of each element. **Figure 3** shows the complete finite element model of a tee section. All *symmetry* model were derivatives of this complete model.

The peak stresses in the tees occurred on the inside face of the elements in the middle of the crotch region. **Tables 2** and **3** show the peak stresses that resulted as a function of the loading and their locations for the size on size and reducing tees, respectively. Also shown in these tables are the FEA SCF values which were determined by dividing the peak stresses in the fittings by the nominal membrane hoop stress in the pipe. **Tables 4** and **5** show the stress concentration factors due to internal pressure for the size on size and reducing tees calculated by using the equations in WRC Bulletin 335 (Equations 10, 11, 12, and 13). Also displayed in these tables are the pertinent tee dimensions and related FEA stress concentration factors.

In viewing the results shown in **Tables 4** and **5**, it would seem that the WRC equations fairly accurately predict the stress concentration factors when compared to the FEA results. The size on size tees with the larger D/t ratios more accurately resemble the SCF values of Equation 10; however, Equation 12 provides the most accurate results for the smaller size on size tee diameters.

In-Plane Bending

The in-plane bending case used a half-symmetry model with approximately 1800 elements. The branch pipe end of the assembly was fixed and the moment was applied to one of the run pipe ends. The moment was selected so that a nominal stress of 20,000 psi would occur in the piping sections. The highest stresses in the size on size tees occurred on the inside of the wall in the saddle region toward the end where the moment has been applied; however, the highest stresses for the reducing tees occurred in a region higher on the branch region of the fitting. This phenomena is important because it would seem that the associated branch in the reducing tee assembly is attempting to "break off" from the rest of the assembly.

The FEA stress concentration factors were calculated by dividing the highest stress in the fitting by the nominal axial stress in the pipe which in this case is 20,000 psi. Because of the general nature of the ANSI B31.8 *i* factors, the results listed in **Tables 2** and **3** show little correlation with their FEA counterparts. Greater accuracy will result with the ANSI values by incorporating a greater number of dimensional factors such as the crotch wall thickness and allowing for different run and branch pipe diameters as is necessary with the reducing tees.

Out-of-plane Bending

The models developed for the out-of-plane bending case utilize full-symmetry with approximately 3600 elements as shown in **Figure 3**. This arrangement alleviates the need for any symmetry boundary conditions. The branch end of the run pipe was fixed and the bending moment was applied using the wagon-wheel approach in the negative Y-direction to one end of the run pipe. This configuration was selected based on previous results which showed that this method caused the highest stresses to appear in the fitting assembly. The magnitude of the moment was selected to induce an axial stress of 10,000 psi in either of the size on size tee pipe sections and 10,000 psi in the branch pipe for the reducing tees.

The highest stresses occur in the saddle region on the outside wall of the fitting for both the size on size and reducing tee types. The SCF results based on these values are presented in **Tables 4** and **5** along with the respective *i* factors. As calculated previously, the SCF values were determined by dividing the peak stress value by the nominal axial stress, which in this case was 10,000 psi. When compared to the *i* factors, these values are quite different, as was the case with the results from the in-plane bending load.

When comparing the in-plane and out-of-plane SCF values for both of the bending cases for the reducing tees, the out-of-plane values are approximately twice the in-plane values. Conversely, the in-plane and out-of-plane SCF values for the size on size tees are very similar. The significance of this is that for the same moment loading conditions, the reducing tees is more likely to exhibit higher stresses or fail when under out-of-plane bending.

EXPERIMENTAL WORK

Burst Test of Elbow Specimen

A 90° long radius 6 inch standard elbow was tested to failure in the Stress Engineering Services Laboratory. The fitting assembly included the elbow fitting, two 6 inch XS pipe attachments (each 30 inches long), and two 6 inch XH caps. The outside diameters and wall thicknesses were measured before testing for prediction purposes and to determine which regions were probable for localized yielding.

Prior to the test, strain gages were installed on the fitting assembly at specified locations. Instrumentation was selected so that strain, internal pressure, and injected water volume could be measured. During testing, pressures were held at selected points for one minute. The process continued until the material in the fitting indicated yield. Once this point was reached, the water pump was allowed to "ramp-up" and increase the water pressure in the system until failure occurred. The failure in the assembly occurred in the crotch region of the elbow and corresponds to the results obtained analytically from the finite element analyses. Initial localized yielding occurred in the crotch region at approximately 4,550 psi with the most significant yielding occurring at 4,800 psi. The burst pressure for the fitting was 7,010 psi.

The primary purpose of the burst test was to determine the burst pressure of the fitting for comparison with existing prediction equations. The Lorenz equation can be rearranged so the pressure can be calculated for a specified stress within the fitting.

$$P = \frac{\sigma t}{1.25 r}$$

The stress value in the above equation could be the stress limits associated with the material properties of the fitting such as the yield and ultimate tensile strengths, although the Lorenz equation may not be valid at stresses above the yield strength. For this testing, a duplicate fitting was purchased and used to obtain the yield and tensile stresses for calculating the yield and burst pressures

The predicted yield and burst pressures were obtained using the Lorenz equation, the material stress values, and the minimum wall thickness measured from the fitting.

$$P_{yield} = \frac{\sigma_{yield} t_{\min}}{1.25 r_i} = \frac{(51{,}000)(.324)}{1.25(3.023)} = 4372 \ p.s.i.$$

$$P_{burst} = \frac{\sigma_{ult} t_{\min}}{1.25 r_i} = \frac{(81{,}400)(.324)}{1.25(3.023)} = 6979 \ p.s.i.$$

The actual yield and burst pressures in the crotch region of the elbow were 4,550 and 7,010 psi, respectively. The actual burst pressure corresponds well to the predicted pressure (within 0.44%). Based on these results, the ultimate tensile strength and the minimum wall thickness appear to be the predominant factors in predicting the accurate burst pressure of the elbow pipe fitting assembly.

Burst Test of Size on Size Tee Specimen

A 6 inch standard size on size tee was tested to failure. The fitting assembly consisted of the tee fitting, three 6 inch XS pipe attachments (each 30 inches long), and two 6 inch XH caps. The dimensions of the fitting were measured prior to testing for use in the burst pressure prediction equations and also to determine probable regions for localized stressing.

Prior to the test, strain gages were installed on the fitting and attached piping as with the elbow test. Instrumentation was selected so that strain, internal pressure, and injected water volume could be measured. During testing pressures were held at selected points for one minute. The process continued until the material of the fitting indicated yield. Once this point was reached, the water pump was allowed to "ramp-up" and increase the pressure in the system until failure occurred. Initial yielding occurred in one of the crotch regions of the fitting which corresponds to the highest stress regions as predicted by FEA. These strain gage rosettes in this region disconnected themselves from the fitting because of the high stresses. A gage located on the back surface of the fitting, was the next region to indicate yielding at approximately 5,000 psi. The fitting eventually failed in this region once the internal pressure reached 9,000 psi. The location of this failure was unexpected; however, measurements indicated that the minimum wall thickness occurred in this region of the fitting.

The tee fitting predicted burst pressure was calculated using Equations 9, 10, and 14 of the WRC Bulletin 335 (9). The material ultimate tensile stress limit obtained from testing of the duplicate fitting and the minimum measured wall thickness were used in these equations.

The burst pressure was determined first by calculating the SCF value as specified by Equation 10. The SCF for the 6 inch fitting was previously found to be 4.573. The next step was to calculate the vessel (pipe with nominal diameter and thickness of the fitting) burst pressure, P_{bv}, using Equation 9 from the WRC Bulletin 335 and the related material values,

$$P_{bv} = \frac{2S_{ult}T}{D} = \frac{2(77{,}200)(0.397)}{6.139} = 9{,}448\ p.s.i.$$

where T is the minimum wall thickness and D is the mean run diameter. Equation 14 from the WRC Bulletin 335 provides the correlation between the fitting and vessel burst pressures,

$$P_b = 2.976(SCF)^{0.8505}P_{bv} = 2.976(4.573)^{-0.8508}(9{,}448) = 7{,}714\ p.s.i.$$

The prediction value can be adjusted as follows using the actual and minimum required tensile strengths from Table 4.1 API SPEC 5L (9). The minimum specified UTS for X52 pipe is 66,000 psi. The adjusted burst pressure, P_{adj}, was calculated using the necessary values,

$$P_{adj} = P_b\left(\frac{\sigma_{ult,act}}{\sigma_{ult}}\right) = 7{,}714\left(\frac{77{,}200}{66{,}000}\right) = 9{,}023\ p.s.i.$$

This predicted value is within 0.26% of the actual 9,000 psi burst pressure. As with the elbow test conclusions, the primary factors in predicting the actual burst pressures of a size on size tee fitting are the ultimate tensile strength and the minimum wall thickness. In this test the location of the minimum wall thickness was also a key factor in pin-pointing where failure was likely to occur.

FINDINGS AND RECOMMENDATIONS

Conclusions Relating to Elbows

The research of this project involved both analytical and experimental work. Several conclusions can be made which may be important for the both the manufacturers and users of elbow pipe fittings.

- Although there are no dimensional requirements in the standards to require that elbows be thicker than the adjoining pipe, the data indicates that commercial elbows are 12 to 70 percent thicker than the adjacent pipe. This indicates that most elbows have some margin of safety, even though stresses in long radius elbows due to pressure are 25 percent greater than the pipe for the same thickness elbow.

- Based on a full-scale test and plastic finite element analysis, it does not appear that yield strength is a significant factor in determining the burst strength of elbows as long as the elbow has adequate ductility. This is true even though the stresses before yield are 25 percent higher in the crotch of the elbow than in the adjacent pipes of the same thickness. This finding has significant implications for consideration of elbows of unknown origin as it indicates that the key variables for burst strength of an elbow is the ultimate strength, the elongation, and the minimum thickness. Two of these can of course be estimated from non-destructive examination of elbows.

- Fatigue due to pressure cycles on elbows should not be a significant problem. Compared to the AGA research on dents, the fatigue of elbows is not nearly as severe as the fatigue of a plain (non-gouged) dent which has a depth of 2 percent of the pipe diameter. **Figure 4** shows the stress per unit pressure for various elbows and for plain dents which are much more severe.

● The B31.8 stress intensification factors appear to be unconservative, particularly for the case of in-plane bending. If the elbow is no thicker than the adjoining pipe, the actual in-plane stress intensification factor can be twice that called out in B31.8. It is possible to have commercial elbows whose *i* factors are similar to those in B31.8. However, this will require elbows that are as much as 50 percent thicker than the adjoining pipe. Much of this discrepancy is due to the fact that *i* factors were determined from fatigue tests on 4 inch size elbows for which good dimensional data was not reported. Also, there is a great concern that when fatigue tests on small sized items are scaled up to large size, the actual fatigue results on large samples will be significantly worse. The saving factor for many situations is that with a buried pipeline the expansion stresses which require consideration of these stress intensification factors are not significant.

● If the users adopted a dimensional rule for long radius elbows that the elbow thickness be at least 25 percent greater than the adjoining pipe, the *i* factor concern would be greatly improved. However, this may not be necessary.

● For inspection, the heat affected zone of the fittings is very critical for out of plane bending loads because the high stress region is close to the elbow end. This means that careful inspection of the fitting to pipe weld is very important.

● Caution should be exercised when material with high SMYS is used specifically for 0.72 design factor areas. Since elbows which are not reinforced have stresses that are 1.25 times the nominal pipe stress, the actual design factor will be at 0.9. For this to be acceptable, care must be given to ensure adequate ductility and construction which is as defect-free as possible.

Conclusions Relating to Tees

Considering the analytical and experimental work performed on the size on size tees, there are several conclusions that can be made:

● The ANSI B31.8 *i* factors do not agree with the SCF values generated by finite elements for in-plane and out-of-plane bending. The B31.8 equations incorporate only a few dimensional values and if these equations are to be applied to a wide range of fitting sizes, then more general equations are necessary which include a larger number of dimensional parameters. In spite of these inadequacies, the gas companies would benefit in requiring that the purchased fittings meet the crotch radius and thickness requirements from Table E1 of B31.8:

$$r_{crotch} \geq D_o/8$$

$$t_{crotch} \geq 1.5\ t_{pipe}$$

● As with the above size on size tee conclusion, the equations which are used to calculate the *i* factors for the reducing tees would benefit considerably by incorporating dimensions such as differentiation in the run and branch diameters. This is crucial, especially when there should be a difference in the stress concentration factors for the reducing and size on size tees.

● Peak stresses in tees due to internal pressure are high (see **Table 2**). They are equivalent to plain dents (without gouges) that are as deep as 5 to 10 percent of the diameter (refer to **Figure 4** for comparison which shows the stress/pressure ratios of 100 are high). This suggestion that is there is a possibility of cyclic pressure, then the fatigue behavior of tees should be considered.

● The experimental burst test provided several significant conclusions. The ability to accurately predict the burst pressure is dependant on the ultimate tensile strength and the minimum wall thickness of the fitting. In this research, the location of the rupture within the fitting corresponded exactly to the location where the minimum wall thickness occurs.

● Based on this research, the equations from the WRC Bulletin 335 provide a better approximation for the burst pressures. The equations are dependant on several fitting dimensions, unlike ANSI B31.8, which allow them to be applied generally to a wide range of sizes.

● The MSS SP-75 standard states that proof testing may serve as evidence for design adequacy. The standard also states that one fitting may be used to qualify another which is neither smaller than one-half nor larger than two times the size of that fitting. Based on this information, users and purchasers should require that manufacturers provide documentation that testing was done and how the testing qualifies the design of those particular fittings. The design qualification should include dimensional requirements which the manufacturer would hold to guarantee that the supplied fittings would have the burst pressures as those predicted by the actual tests.

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TABLE 1
STRESS CONCENTRATION AND FLEXIBILITY FACTORS 90° LONG RADIUS ELBOWS UNDER BENDING MOMENT LOADING

				In-Plane Bending	Out-of-plane Bending	Flexibility Factor
Nominal Pipe (inches)	Pipe Thickness (inches)	Reinf. Type	Crotch Thickness (inches)	FEA/ ANSI B31.8	FEA/ ANSI B31.8	FEA/ ANSI B31.8
6	0.280	Manufacture	0.314	3.83 / 2.18	2.69 / 1.82	5.24 / 6.25
8	0.500	Manufacture	0.521	3.41 / 1.75	2.41 / 1.46	3.88 / 4.46
10	0.500	Manufacture	0.534	3.80 / 2.05	2.65 / 1.71	4.91 / 5.68
12	0.375	Manufacture	0.412	5.17 / 2.79	3.57 / 2.33	7.82 / 9.04
6	0.280	1.25	0.350	3.18 / 2.18	2.28 / 1.82	4.79 / 6.25
12	0.375	1.25	0.469	4.19 / 1.75	2.93 / 1.46	7.19 / 4.46
6	0.432	N/R	0.432	3.17 / 1.64	2.21 / 1.37	3.59 / 4.05
12	0.500	N/R	0.500	4.87 / 2.31	3.37 / 1.93	6.25 / 6.78
8	0.322	ANR	0.403	3.54 / 2.34	2.40 / 1.95	7.31 / 6.93
8	0.875	N/R	0.875	2.11 / 1.20	1.61 / 1.00	2.26 / 2.55
24	0.25	N/R	0.25	11.36 / 5.64	6.36 / 4.70	24.83 / 25.85
24	0.25	ANR	0.313	9.52 / 5.64	5.34 / 4.70	22.69 / 25.85
24	0.25	1.25	0.313	8.37 / 5.64	4.52 / 4.70	21.16 / 25.85
24	0.25	Manufacture	0.363	6.75 / 5.64	3.87 / 4.70	18.85 / 25.85
24	1.50	N/R	1.50	3.28 / 1.59	2.29 / 1.32	3.34 / 3.87
24	1.50	1.25	1.875	2.27 / 1.59	1.69 / 1.32	2.64 / 3.87
36	1.44	N/R	1.44	4.60 / 2.21	3.02 / 1.84	5.73 / 6.34
36	1.44	1.25	1.80	3.28 / 2.21	2.22 / 1.84	4.64 / 6.34
16	0.25	N/R	0.25	8.83 / 4.27	4.95 / 3.56	16.16 / 17.05
16	0.25	1.25	0.313	6.44 / 4.27	3.88 / 3.56	13.57 / 17.06

Notes:
1. Reinforcement type *1.25* indicates that the total elbow has a thickness of 1.25 times the pipe thickness.
2. Reinforcement type *Manufacture* indicates that elbows dimensions are based on actual measured values.
3. Reinforcement type *ANR* indicates the crotch region of the elbow has been reinforced 1.25 times the pipe thickness.
4. Reinforcement type *N/R* indicates that no reinforcement in the elbow is used, thus the thickness equals the attached pipe thickness.

TABLE 2
SIZE ON SIZE TEES FINITE ELEMENT RESULTS AND S.C.F. VALUES

Tee Size (inches)	Loading Case	Peak Stress (psi)	Location of Stress	Nominal Stress (psi)	FEA SCF	ANSI SCF i_i, i_o
4X4, XH	Pressure	27,083	Crotch (inside)	5,091	5.32	
	In-plane	83,271	Saddle (inside)	20,000	4.16	1.10
	Out-of-plane	35,447	Saddle (outside)	10,000	3.54	1.13
6X6, STD	Pressure	37,128	Crotch (inside)	6,245	5.94	
	In-plane	51,516	Saddle (inside)	20,000	2.58	1.52
	Out-of-plane	25,608	Saddle (outside)	10,000	2.56	1.69
8X8, XH	Pressure	30,476	Crotch (inside)	4,828	6.31	
	In-plane	85,039	Saddle (inside)	20,000	4.25	1.29
	Out-of-plane	36,171	Saddle (outside)	10,000	3.62	1.38
12X12, XH	Pressure	66,126	Crotch (inside)	12,706	5.20	
	In-plane	101,240	Saddle (inside)	20,000	5.06	1.59
	Out-of-plane	50,746	Saddle (outside)	10,000	5.07	1.78
24 X 24	Pressure	77,570	Crotch (inside)	18,700	4.15	
	In-plane	38,880	Saddle (inside)	10,000	3.89	2.02
	Out-of-plane	50,700	Saddle (outside)	10,000	5.07	2.36
24 X 24	Pressure	126,740	Crotch (inside)	47,500	2.67	
	In-plane	35,458	Saddle (inside)	10,000	3.55	3.55
	Out-of-plane	33,622	Saddle (outside)	10,000	3.36	4.40
24 X 24	Pressure	48,755	Crotch (inside)	11,500	4.24	
	In-plane	37,248	Saddle (inside)	10,000	3.72	1.53
	Out-of-plane	37,306	Saddle (outside)	10,000	3.73	1.71
36 X 36	Pressure	157,840	Crotch (inside)	57,190	2.76	
	In-plane	34,582	Saddle (inside)	10,000	3.46	3.98
	Out-of-plane	36,931	Saddle (outside)	10,000	3.69	4.98
36 X 36	Pressure	42,813	Crotch (inside)	11,983	3.57	
	In-plane	36,920	Saddle (inside)	10,000	3.69	1.57
	Out-of-plane	40,410	Saddle (outside)	10,000	4.04	1.76
36 X 36	Pressure	106,710	Crotch (inside)	35,000	3.05	
	In-plane	35,459	Saddle (inside)	10,000	3.54	2.97
	Out-of-plane	42,855	Saddle (outside)	10,000	4.29	3.62

TABLE 3
REDUCING TEES
FINITE ELEMENT RESULTS AND S.C.F. VALUES

Tee Size (inches)	Loading Case	Peak Stress (psi)	Location of Stress	Nominal Stress (psi)	FEA SCF	ANSI SCF i_i, i_o
8X3, STD	Pressure	25,393	Crotch (inside)	7,079	3.45	
	In-plane	78,884	Saddle (inside)	20,000	3.94	1.63
	Out-of-plane	72,785	Saddle (outside)	10,000	7.28	1.84
8X6, STD	Pressure	29,200	Crotch (inside)	6,834	3.59	
	In-plane	44,510	Saddle (inside)	20,000	2.23	1.63
	Out-of-plane	57,531	Saddle (outside)	10,000	5.73	1.84
10X4, STD	Pressure	24,044	Crotch (inside)	6,972	4.05	
	In-plane	62,301	Saddle (inside)	20,000	3.12	1.73
	Out-of-plane	60,288	Saddle (outside)	10,000	6.03	1.97
12X6, XH	Pressure	25,349	Crotch (inside)	6,257	4.27	
	In-plane	54,081	Saddle (inside)	20,000	2.70	1.59
	Out-of-plane	60,725	Saddle (outside)	10,000	6.07	1.78

TABLE 4
SIZE ON SIZE TEE DIMENSIONS AND STRESS CONCENTRATION FACTORS DUE TO INTERNAL PRESSURE FOR FEA AND WRC BULLETIN 335

Tee	D=d	T=t	$t=t_n$	r_2	λ	FEA SCF	Eq. 10	Eq. 11	Eq. 12	Eq. 13
4 X 4	4.256	.337	.470	.875	3.55	5.32	3.56	3.96	4.47	4.22
6 X 6	6.248	.280	.420	1.00	4.76	5.94	3.83	4.58	5.94	4.98
8 X 8	8.102	.500	.850	1.25	4.03	6.31	3.72	4.22	5.37	4.52
12 X 12	12.706	.500	.712	1.75	5.04	5.20	3.96	4.71	5.70	5.15
24 X 24	23.375	.625	.0979	4.88	6.12	4.15	3.6	5.2	6.6	5.8
24 X 24	23.75	3.25	0.500	3.0	9.75	2.67	3.8	6.6	9.7	8.1
24 X 24	23.3	1.0	1.188	3.0	4.80	4.24	4.0	4.6	5.5	5.0
36 X 36	35.688	.312	.625	4.5	10.7	2.76	3.8	6.9	10.6	8.7
36 X 36	34.588	1.442	1.75	4.5	4.90	3.57	4.0	4.6	5.6	5.1
36 X 36	35.5	.500	.89	6.0	8.43	3.05	3.7	6.1	8.6	7.3

TABLE 5
REDUCING TEE DIMENSIONS AND STRESS CONCENTRATION FACTORS DUE TO INTERNAL PRESSURE FOR FEA AND WRC BULLETIN 335

Tee	D	d	T	t	t_n	r_2	λ	FEA SCF	Eq. 10	Eq. 11	Eq. 12	Eq. 13
8 X 3	8.304	3.284	.322	.216	.489	.63	2.01	3.59	2.96	3.64	4.21	3.54
8 X 6	8.304	6.435	.322	.280	.397	1.0	3.88	4.27	3.69	4.44	5.10	4.57
10 X 4	10.386	4.263	.500	.237	.447	.69	1.87	3.45	3.43	3.79	3.75	3.89
12 X 6	12.25	6.193	.500	.432	.621	1.0	2.50	4.05	3.35	3.57	3.75	3.69

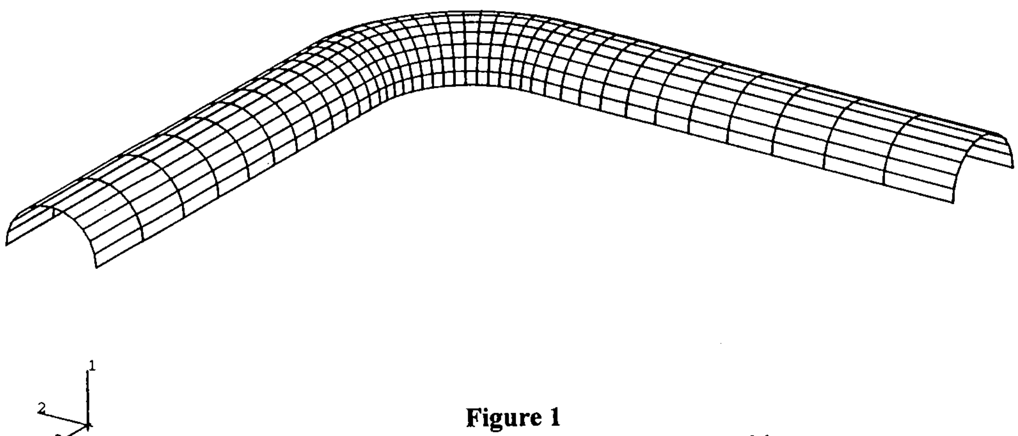

Figure 1
Finite Element Model of Elbow Fitting Assembly

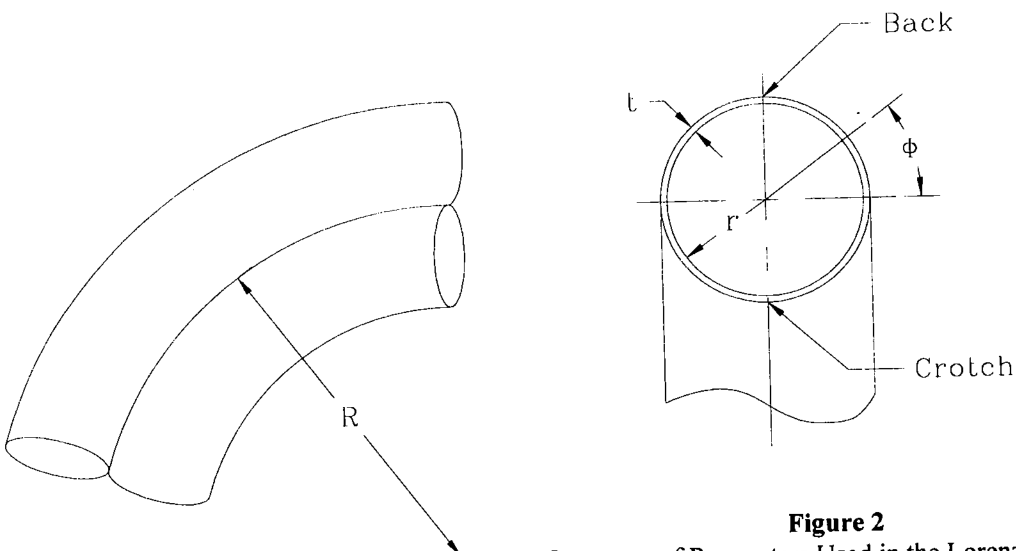

Figure 2
Geometry of Parameters Used in the Lorenz Equation

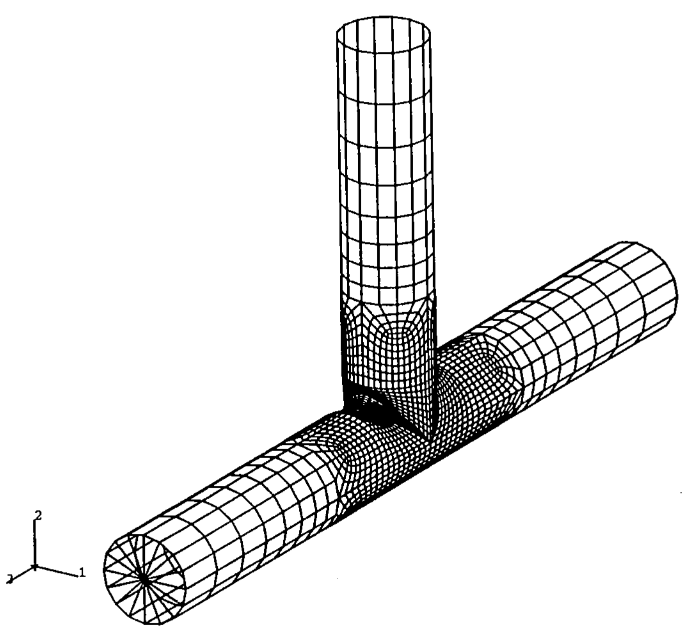

Figure 4

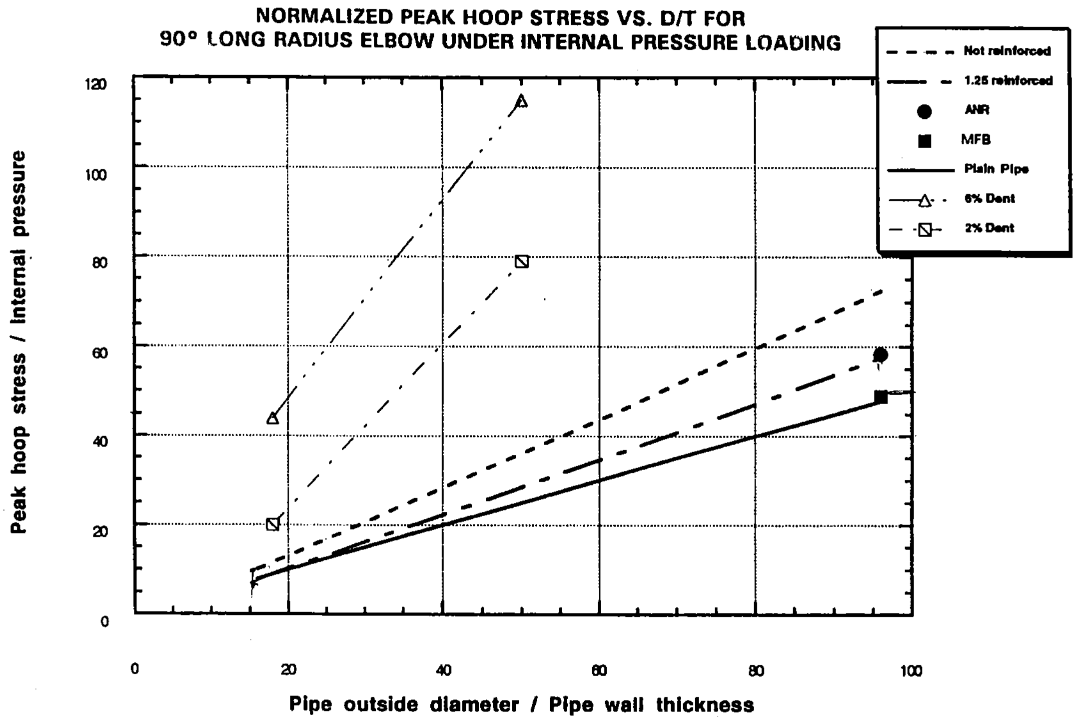

Figure 5