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RATIONAL STRESS LIMITS AND LOAD FACTORS FOR FINITE ELEMENT ANALYSES IN PIPELINE APPLICATIONS PART III – ELASTIC-PLASTIC LOAD FACTOR DEVELOPMENT

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ABSTRACT

In this paper, a methodology is presented to develop load factors for use in elastic-plastic assessments of pipelines and their components. The load factors are based on the pipe material properties and the ASME pipeline code's design margin for the service and location of the pipeline installation [1, 2]. These codes are recognized by 49 CFR 192 and 195 [3, 4].

Minimum required load factors for internal pressure loads can be derived analytically based on design equations from the ASME B31 piping codes and minimum material requirements for API 5L line pipe [6]. Once the load factor is established for a particular case, the elastic-plastic methodology may be used in the Finite Element Analysis (FEA) of pipelines and related components. This methodology is particularly useful in the assessment of existing systems when linear elastic numerical analysis shows that local stresses may exceed the elastic design limits.

Two case studies are presented showing analyses performed with Abaqus [5], a commercial, general purpose FEA software package. The first case study provides an assessment of a large diameter elbow where the stress on the outer fibers of the intrados exceeded the longitudinal stress limits from B31.8. The second case study examines an assessment of a tee connection where the stresses on the ID exceeded the yield strength of the component. In addition to the case studies, the paper also presents the results of a full-scale test that demonstrated what margin was present when the numerical calculations were based on specified minimum properties.

This paper is not intended to revise or replace any provision of B31.4 and/or B31.8 [1, 2]. Instead, it provides the means for calculating load factors that can be used with an elastic-plastic analysis approach in a manner that provides the same design margins as the ASME B31 codes. The approach described in this paper is intended for use in the detailed FEA of pipelines and their associated components.

INTRODUCTION

ASME B31.4 and B31.8 provide simplified design equations for pressure piping [1, 2]. These equations give the design pressure such that the hoop stress is nominally limited to a certain portion of the specified minimum yield strength of the pipe material (S_y). Additional equations from the ASME codes permit the calculated longitudinal and combined stresses to be some fraction of S_y. These equations can be applied to straight segments of pipe with relative ease. However, many piping components have complex shapes or non-linear stress-displacement relationships that require the use of FEA to precisely calculate their state of stress. Typical examples may include tees, elbows, or wyes. The current pipeline codes do not specify a method or outline an approach for conducting FEA. Instead they refer to the ASME B&PV code, Section VIII, Division 2 (Division 2).

When an analysis approach according to Division 2 [7] is used to assess a pipeline component, several methodologies are available including linear, limit-load, and elastic-plastic. This paper focuses on the elastic-plastic assessment methodology. When performing an elastic-plastic assessment, the following question must be answered. What is the appropriate load factor for an elastic-plastic assessment of a pipeline component? The load factor should, at a minimum, provide an equivalent design margin to any adjacent components which may have been designed according to the ASME B31 piping codes [1, 2].

For a pipeline component such as an elbow or tee, it may be necessary to perform a structural integrity assessment using FEA. This paper describes the methodology for analytically identifying an appropriate case specific load factor which can be used in an elastic-plastic FEA. Two case studies utilizing this approach are provided in the paper as well as a comparison to a full-scale burst test.

NOMENCLATURE

D	nominal outside diameter of pipe
D_i	inside diameter
D_o	outside diameter
E	longitudinal joint factor
F	design factor
F_L	load factor for pressure, limit-load analysis
F_P	load factor for pressure, elastic-plastic analysis
P	design pressure
$R=S_y/S_u$	engineering yield to engineering tensile ratio
S_u	specified ultimate tensile strength
S_y	specified minimum yield strength
t	nominal wall thickness
T	temperature derating factor
$Y=Do/D_i$	diameter ratio
$\sigma_{m\_eq\_d}$	von Mises equivalent membrane stress at design pressure

CASE STUDY BACKGROUND

The usefulness of an elastic-plastic analysis methodology is demonstrated in the two case studies described in this paper. In both case studies, the systems under consideration were already installed and had been previously designed using conventional analysis methods including stress intensification factors (SIFs) and flexibility factors.

The first case study is based on a large diameter elbow that was reassessed as part of an integrity management review. During the course of the review, updated soil properties resulted in higher stresses in the elbows. In particular, the stresses on the intrados of the elbow were noted to slightly exceed the values permitted in B31.8 [2]. An elastic-plastic analysis based on measured properties was used to demonstrate that despite the high stresses on the intrados, the elbows had a design margin that was equivalent to the adjacent line pipe.

The second case study examines a tee and a reducer in a subsea application. The initial design used SIFs combined with a typical beam element analysis. A detailed local analysis conducted after the system was installed determined that the peak stresses on the ID of the tee were higher than the original SIFs predicted. In fact, the detailed analysis showed that the ID stresses exceeded yield. Again, an elastic-plastic analysis based on minimum specified properties was used to demonstrate that the fittings had design margins that were equivalent to the adjacent line pipe, despite the higher stresses on the ID.

Before presenting the results of both case studies, it is necessary to establish the methodology for determining the load factors. It is important to note that elastic-plastic assessments must use a case specific load factor that is dependent on the geometry, material properties, and original design factor of the pipeline in question. This is in contrast to the single load factors that may be used in design with linear elastic analysis.

It should also be noted that that the authors are not suggesting that the routine design of piping systems or components utilize elastic-plastic assessments in lieu of traditional elastic design approaches. Rather, this methodology is presented as a means for assessing challenging pipeline components and demonstrating that they have equivalent design margins to the adjacent line pipe.

THEORETICAL DEVELOPMENT

The design pressure for gas transmission piping systems is based on the maximum principal stress theory using Barlow's equation for thin-walled pipe. The internal design pressure from B31.8 [2] is written as

$$P = \frac{2S_y t}{D} FET \tag{1}$$

A similar equation may be found in B31.4 [1] although it is arranged in a slightly different manner. Equation (1) is used to calculate the design pressure for a pipeline based on its wall thickness, diameter, location, and design temperature. Additional equations addressing the longitudinal and combined stresses in piping systems can be found in B31.8 [2] for both unrestrained and restrained pipe. In the case of combined stresses for restrained pipe, B31.8 [2] specifically states that the equations presented only apply to straight sections of pipe. Furthermore, the B31.8 code [2] states that it does not fully address the maximum allowable stresses for local stresses that might occur at structural discontinuities. These last two points highlight the need for guidance when using finite element techniques on pipeline components.

Hoop stresses in B31.8 are governed by a design factor, "*F,*" that may vary from 0.4 to 0.8 [2]. The design factor is 0.72 for many transmission systems in the United States. For longitudinal and combined stresses, a separate factor, *k*, is used that may range from 0.75 to 1. When an elastic-plastic analysis approach is used to evaluate the integrity of a component for internal pressure, a similar single value load factor is needed. Such a load factor should provide a design margin equivalent to the one provided by B31.8 [2].

Using the Lame` solutions [8], it can be shown that the von Mises equivalent membrane stress at the design pressure for closed and open end conditions (or with and without pressure end load (PEL)) is given by

$$\sigma_{m\_eq\_d} = \begin{cases} \dfrac{\sqrt{3}}{Y+1} FS_y & \text{(with PEL or closed end)} \\ \dfrac{\left(1+3Y^2\right)^{1/2}}{Y\left(Y+1\right)} FS_y & \text{(without PEL or open end)} \end{cases} \quad (2)$$

If Y is taken as 1 (i.e., thin-walled pipe), then equation (2) reduces to

$$\sigma_{m\_eq\_d} = \begin{cases} \dfrac{\sqrt{3}}{2} FS_y & \text{(with PEL or closed end)} \\ FS_y & \text{(without PEL or open end)} \end{cases} \quad (3)$$

If it is assumed that plastic collapse occurs whenever the equivalent von Mises stress reaches the specified ultimate tensile strength of the material, the load factor for plastic collapse can be defined as $F_P = S_u / \sigma_{m\_eq\_d}$. It should be pointed out that the term "plastic collapse" is used to maintain consistency with the nomenclature from Division 2. For pipeline applications, the term plastic collapse can be considered to be synonymous with burst as this is the typical failure mode for pipelines. If a substitution for the plastic collapse factor, F_P, is made in Equation (3), the following relationship can be derived:

$$\frac{S_u}{F_P} = \begin{cases} \dfrac{\sqrt{3}}{2} FS_y & \text{(with PEL or closed end)} \\ FS_y & \text{(without PEL or open end)} \end{cases} \quad (4)$$

If equation (4) is rearranged to solve for F_P, and the engineering yield to tensile ratio, $R = S_y/S_u$, is substituted into the equation, the final plastic collapse factors become

$$F_P = \begin{cases} \dfrac{1.15}{FR} & \text{(with PEL or closed end)} \\ \dfrac{1}{FR} & \text{(without PEL or open end)} \end{cases} \quad (5)$$

The relationship in equation (5) is convenient for thin-wall pipe applications as it expresses the plastic collapse factor as a function of the design factor and the specified minimum material properties of the pipe, which can be obtained from material specifications such as API 5L [6]. For example, API 5L Grade X70 material (R = 0.85) with a design factor of 0.72 will have a plastic collapse factor of 1.88 for closed end conditions or 1.63 for open end conditions. For comparison, Division 2 [7] uses a factor of 2.4 for pressure and dead loads in an elastic-plastic analysis (Table 5.5 in [7]).

Elastic-plastic load factors can be derived for other API 5L materials and load factors as shown in **Table 1** and **Table 2** for thin-wall applications. It is noteworthy that larger factors are required for lower yield strength materials due to the lower yield to tensile ratios.

ANALYSIS APPROACH AND PROCEDURE

Division 2 describes three separate analysis techniques which are linear elastic, limit-load, and elastic-plastic. Division 2 provides guidance on how each analysis is to be performed. A brief summary of the elastic-plastic approach is provided here. An elastic-plastic FEA according to Division 2 [7] has the following features:

- Mesh refinement around areas of stress and strain concentrations
- The material model is elastic-plastic and may include hardening
- The von Mises yield function and associated flow rule should be utilized
- The effects of non-linear geometry are included in the analysis (i.e., large displacement theory)
- The numerical model must achieve convergence with a stable solution at the required load factor

The material model used in the elastic-plastic analysis should be input using true-stress, true-strain values. Most materials do not have true-stress, true-strain data readily available. Therefore, the material models may be obtained by converting material test data expressed as engineering stress and engineering strain according to constant volume relationships. Alternatively, the true-stress, true-strain material curve models from Division 2, Appendix 3-D can be used [7].

A detailed procedure for an elastic-plastic analysis is as follows:

Step-1: For thin-wall applications, use equation (5) to determine the plastic load factor based on the von Mises yield criterion considering the material properties and end conditions for the system being analyzed. For thick-wall applications, use equation (2) and follow the same substitutions to solve for F_p.

Step-2: Develop a finite element model with an elastic-plastic material model and large displacement theory (i.e., non-linear geometry). Actual test data may be used for the material model, or the specified minimum properties may be used with the material curves from Division 2 [7].

Step-3: Apply factored pressure as well as other applicable loads that may result from internal pressure.

Step-4: Run the required load cases and determine the burst pressure. This step can be accomplished by incrementally increasing the loads until the maximum pressure has been identified (i.e., burst pressure in most cases).

The plastic collapse load (i.e., burst for pipeline applications) is the load that causes overall structural instability in the model. The collapse load is indicated by the inability of the model to achieve equilibrium with small increases in load. A load-displacement curve at critical locations will usually demonstrate the progress of the structural response. A plot of the peak plastic strains versus the load may also be useful in evaluating the structural response. Furthermore, Riks analysis methods in Abaqus (or similar path dependent solution methods in other software packages) can also be used to ensure that the ultimate load has been reached.

It should be pointed out that Division 2 requires additional checks for local failure and ratcheting which are not addressed in this paper. It is recommended that an experienced engineer evaluate the application and ensure that these checks can be satisfied.

In order to illustrate how the elastic-plastic analysis approach can be used, two case studies are presented in the next section. In addition, a full-scale burst test is presented to demonstrate the conservatism that was present in the case study when the specified minimum properties for line pipe were used in the assessment.

CASE STUDIES

I. Re-Assessment of a Large Diameter Elbow

The first case study examines a large diameter elbow in a natural gas pipeline. The pipeline in question was constructed from API 5L, Grade X70 material with a design factor, *F*, of 0.8, and a design pressure of 1,440 psi (9.9 MPa). The diameter of the pipe was 36 inches (914 mm) with a nominal wall thickness of 0.465 inches (11.81 mm). The elbow in question was a 3D forged elbow with a thickness of 0.59 inches (15 mm). The adjacent transition spool pieces were 0.54 inches (13.7 mm) thick. A graphical image of the elbow configuration is shown in **Figure 1**.

The need for an elastic-plastic analysis was identified when the elbow in question was determined to be overstressed based on a conventional linear elastic analysis. The maximum combined stress at the intrados of the elbow was 65.2 ksi (450 Mpa), or 93% of S_y. This combined stress exceeded the allowable limits for the elbow, which were taken as 90% of S_y. Therefore, an elastic-plastic analysis methodology was used to evaluate the integrity of the elbow.

This case study demonstrates the need for rational design factors when using the elastic-plastic methodology to assess components in systems designed according to B31.8 [2]. If an elastic-plastic analysis is performed using the specified factor of 2.4 from Division 2, the analysis will fail to converge before the desired load factor is reached because the burst capacity of the line pipe will be exceeded. Herein lies the problem – if the line pipe cannot reach the specified factor of 2.4, then the adjacent elbow cannot reasonably be expected to meet this requirement unless it is 60% or more thicker than the line pipe (based on the ratio of 2.4 to the required load factor of 1.47 from **Table 1**). One course of action could be to place a specification break in the pipeline and design the elbow to a different code such as Division 2 while the pipeline is designed to B31.8 [2]. However, this will produce an elbow that may be overly conservative compared to the adjacent line pipe and unnecessarily thick. Overly thick elbows are not desirable for a number of reasons including pig-ability and the need for carefully designed transitions. It is more reasonable to identify a rational load factor, that when combined with an established methodology, will ensure that the elbow meets or exceeds the strength of the adjacent line pipe.

An elastic-plastic FEA model of the elbow was built and analyzed with reduced integration shell elements. An image of the model with the mesh density is shown in **Figure 2**. Elastic-plastic material properties were assigned to the transition spool and elbow components. **Figure 3** provides a plot of the true stress-strain curves for the spool and elbow materials, which were based on measured properties. The line pipe included in the model was assigned elastic properties. The purpose for using elastic properties in the line pipe is two-fold. First, the focus of the analysis was on the elbow component in this case study. Assigning elastic-plastic properties to the line pipe would result in the convergence of the analysis being controlled by the line pipe rather than the elbow or spool pieces. Second, the line pipe was included in the analysis only to transfer external loads into the elbow and ensure that the loads were applied at a sufficient distance from the elbow such that they did not impact the results.

The analysis included internal pressure, temperature, and external loads. The external loads were based on a global model of the pipeline in question and included axial loads and bending moments. The loads were applied as shown in **Figure 4**. One end of the pipe was fixed, and external loads from the global model were applied on the opposite end. The applied temperature change was 97.5°F (36.4°C). The internal pressure was 1,440 psi (9.9 MPa). An elastic-plastic design factor of 1.47 was selected for this analysis since it is a buried pipeline.

However, the input files were specified to incrementally increase the loads to twice the design values if convergence could be achieved.

The model failed to converge at a load factor of 1.87, exceeding the required load factor of 1.47 for this case study. A contour plot of the plastic strains on the displaced configuration of the model is provided in **Figure 5**. The displaced shape of the model is magnified by a factor of 20. Failure occurs in the elbow as evidenced by the high plastic strains in the area and the displaced configuration. A plot of the load factor vs displacement for the node where the loads are applied is shown in **Figure 6**. **Figure 6** also plots the peak plastic strains from the intrados of the elbow against the load factor. This plot confirms that the elbow has reached the burst pressure as the strains in this region are rapidly increasing with small increases in the load factor.

The analysis demonstrated that the elbow had a design margin that exceeded the adjacent line pipe which was designed according to B31.8 [2], and was suitable for the intended service conditions.

II. Re-Assessment of a Tee and Reducer

The second case study involves a tee fitting with a reducer in a subsea application. An image of the tee and reducer with the nominal dimensions noted is shown in **Figure 7**. The fittings were designed for a differential pressure of 10,000 psi (69 MPa) with both internal and external pressures acting on the component. The tee was constructed from a material with S_y=75 ksi (517 MPa) and S_u=95 ksi (655 MPa) while the reducer had a S_y=65 ksi (448 MPa) and S_u=77 ksi (531 MPa). A graphical presentation of the material properties used in the analysis is shown in **Figure 8**. The design factor for the adjacent pipe material was 0.72.

An elastic-plastic analysis was needed because linear elastic analyses showed high local stresses exceeding the ultimate strength of the material on the inside corner of the tee as shown in **Figure 9**. Linearized stresses at these locations confirmed the membrane plus bending stresses were above typical allowable stresses. As a result, an elastic-plastic analysis was used to confirm the integrity of the component.

An elastic-plastic FEA model of the tee and reducer was built. The model was constructed from 8-noded, reduced integration, solid elements. Material properties were generated based on the specified minimum properties and the methodology from Division 2.

The analysis included internal pressure, external pressure, and external loads. The external loads were based on a global model of the piping system and included axial loads and bending moments. The loads were applied as shown in **Figure 10**. The upstream end of the tee was fixed while the downstream end and branch connection had applied external loads. The internal pressure and external pressure were applied with a resulting differential of 10,000 psi (69 MPa) based on the operating conditions and water depth of the system. This case study required thick-wall formulation for the development of the load factors. The design factor was based on the adjacent line pipe with a Y-value of 1.28, an R-value of 0.79, a design factor of 0.72, and closed end conditions. If these values are used with Equation (2) and the same methodology presented in the prior section is followed, the resulting load factor is 2.32 for this case study with closed end conditions.

The tee and reducer model reached a load factor of 2.40 before the model failed to converge. A plot of the load vs displacement for the model is shown in **Figure 11**. Failure occurred due to the thinner portions of the tee reaching their capacity (i.e., a burst failure). The analysis confirmed that the peak stresses at the ID corners of the tee did not indicate that the tee failed to meet requirements. In addition, the analysis confirmed that the tee had a design margin that exceeded the adjacent line pipe and was suitable for the intended service conditions.

FULL-SCALE TEST COMPARISON

The approach documented in this paper was compared to a full-scale burst test on a piece of straight pipe. The demonstration test was conducted on 36 inch (914 mm) diameter pipe with a wall thickness of 0.5 inches (12.7 mm). The pipe was manufactured from X70 material. For a component on a pipeline with a design factor of 0.72, the required elastic-plastic load factor would be 1.88 for closed end conditions. It should be noted that the authors are not suggesting that the design of line pipe utilize elastic-plastic assessments. Rather, this simplified example is intended to demonstrate that the basic methodology provides the same design margins as the line pipe required by the ASME codes [1, 2].

An elastic-plastic analysis of a straight segment of pipe using axisymmetric elements was conducted, and the analysis provided a burst pressure of 2,638 psi (18.2 MPa). Accounting for the elastic-plastic load factor, the resulting design pressure would be 1403 psi (9.67 MPa). It is noted that the design pressure based on a hoop stress of 72% S_y from B31.8 [2] would be 1400 psi (9.65 MPa) confirming that the elastic-plastic approach provides equivalent results to a design according to B31.8.

An image of the full-scale test sample is shown in **Figure 12**. Biaxial strain gages were used to measure the strains at four locations on the sample. The internal pressures were monitored with pressure transducers. The pressure in the sample was incrementally increased with five minute holds specified at pressures corresponding to 72% S_y and 100% S_y.

The sample failed with an internal pressure of 2,966 psi. Images of the failed sample are shown in **Figure 13**. The failure occurred in the pipe body and was ductile in nature. A plot of the internal pressure verse the hoop strain from the test is shown in **Figure 14**. In the test, the strain gage failed at 18,000 με with an internal pressure of 2,820 psi (19.4 MPa). Although the actual burst pressure occurred at a pressure of 2,966 psi (20.4 MPa) after the strain gage failed, the plot confirms the behavior of the analytical models.

The full-scale test confirms that the ultimate capacity predicted by the elastic-plastic analysis based on specified minimum properties is reasonable and equivalent to B31.8. In addition, the test demonstrated what margin remained for the example at hand. In the case of the full-scale test presented in this paper, the test results were 12% higher than the predicted burst pressure from the analysis. It should be noted that this observed margin will vary and depends on the actual pipe properties and geometric tolerances.

CONCLUSIONS

This paper has presented a methodology for determining load factors for elastic-plastic analysis of components designed according to B31.8 [2]. The elastic-plastic factor is case specific and depends on the material design factor of the adjacent line pipe as well as the specified minimum yield and ultimate strengths of the line pipe material. It has been shown that the load factor, F_P, has equivalent design margin to line pipe designed according to B31.8.

A general procedure for performing elastic-plastic analysis of pipeline components has been presented. Two case studies were presented in this paper demonstrating the usage of the proposed load factors. In addition, a full-scale burst test was presented that confirms the analysis provides similar design margins to B31.8 [2], and the case under consideration had some additional margin since the actual properties exceeded the specified minimum properties.

Finally, this paper is not intended to revise or replace any provision of the ASME B31 piping codes. In addition, the use of an elastic-plastic analysis methodology requires the engineer to be familiar with fracture toughness of materials and/or the serviceability limit states that may control a design. This paper is intended to provide pipeline engineers with a rational elastic-plastic load factor that may be used with detailed finite element assessment of pipeline components.

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Table 1: Plastic Load Factors for Internal Pressure in Thin-Wall Applications with Open End Conditions

Material Grade	Design Factor, *F*				
	0.4	**0.5**	**0.6**	**0.72**	**0.8**
Grade B	4.24	3.39	2.83	2.36	2.12
X42	3.57	2.86	2.38	1.99	1.79
X46	3.40	2.72	2.27	1.89	1.70
X52	3.19	2.56	2.13	1.77	1.60
X60	3.13	2.50	2.09	1.74	1.57
X65	2.97	2.38	1.98	1.65	1.49
X70	2.94	2.35	1.96	1.63	1.47

Table 2: Plastic Load Factors for Internal Pressure in Thin-Wall Applications with Closed End Conditions

Material Grade	Design Factor, F				
	0.4	**0.5**	**0.6**	**0.72**	**0.8**
Grade B	4.88	3.90	3.25	2.71	2.44
X42	4.11	3.29	2.74	2.28	2.06
X46	3.91	3.13	2.61	2.17	1.95
X52	3.67	2.94	2.45	2.04	1.84
X60	3.60	2.88	2.40	2.00	1.80
X65	3.42	2.73	2.28	1.90	1.71
X70	3.38	2.71	2.25	1.88	1.69

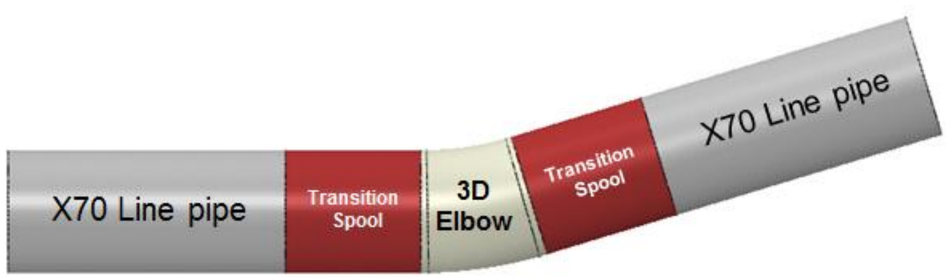

Figure 1: Elbow Configuration

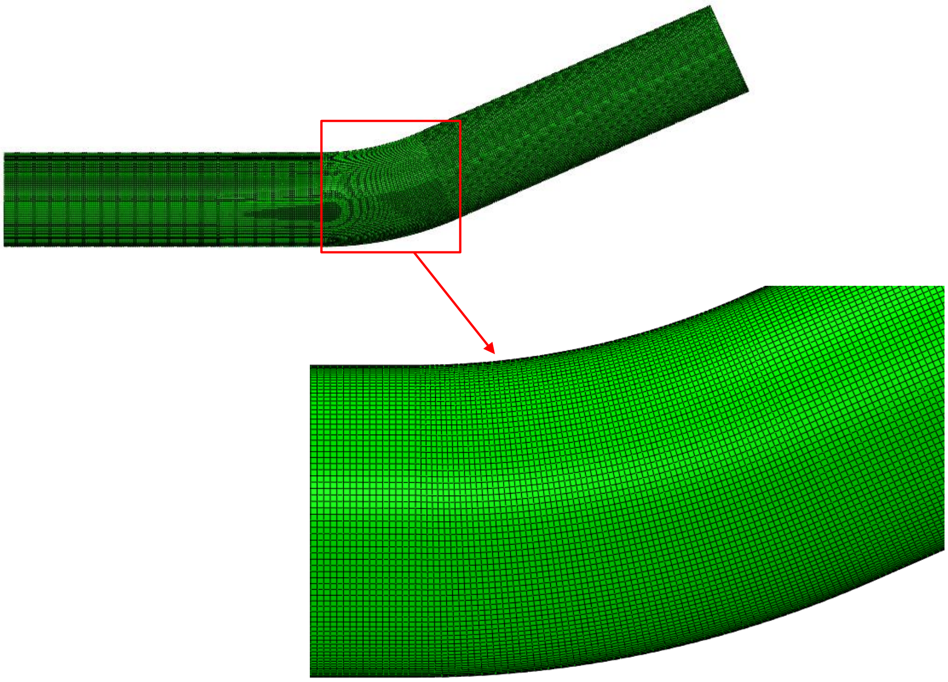

Figure 2: Elbow Mesh Density

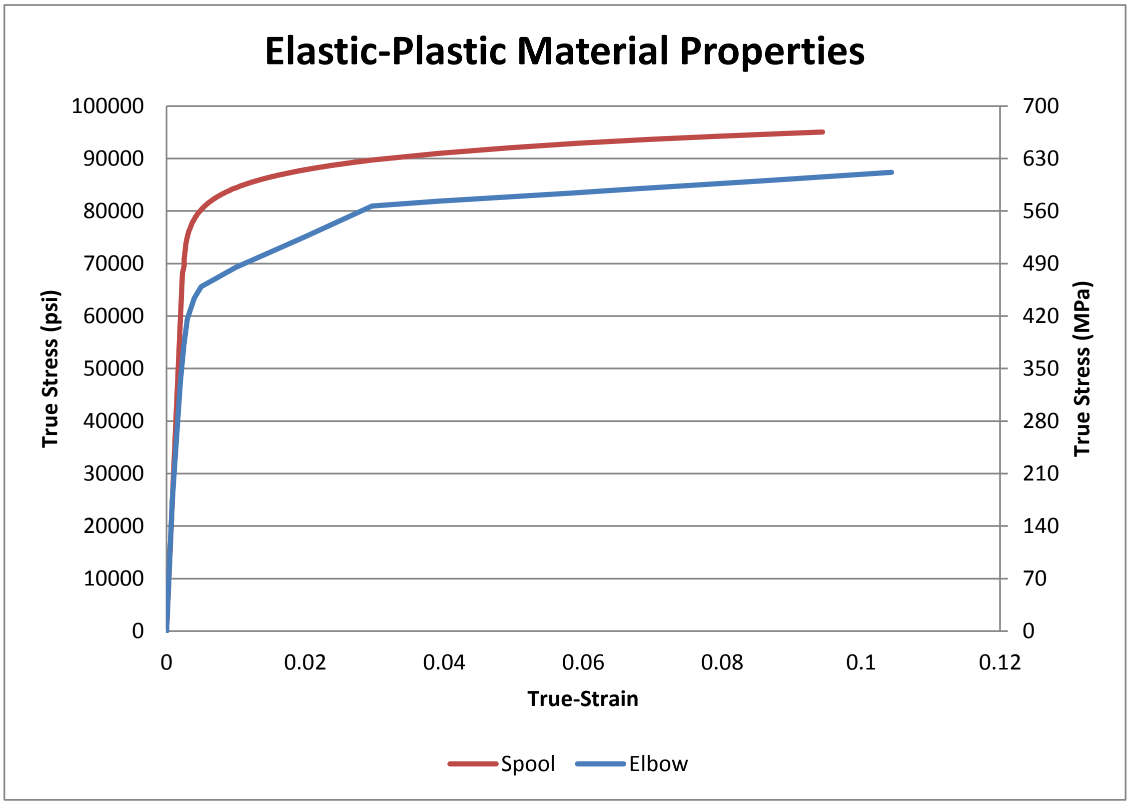

Figure 3: True Stress-Strain Material Properties

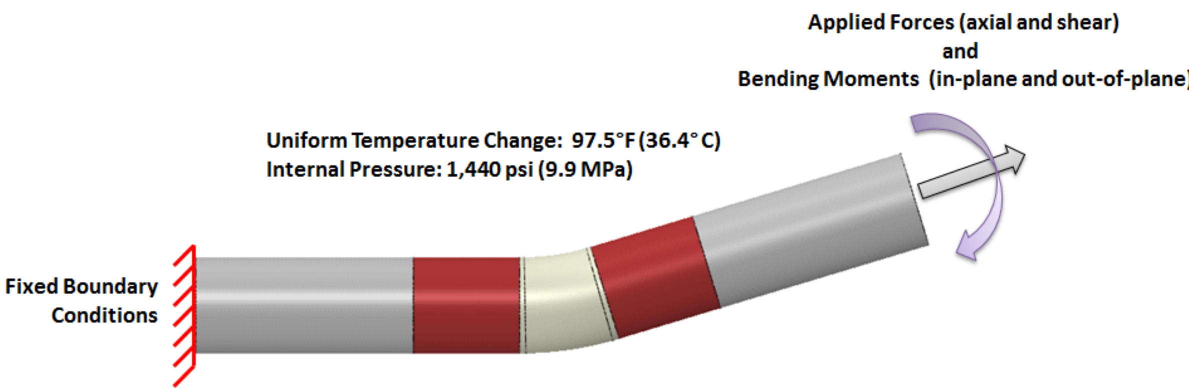

Figure 4: Applied Load and Boundary Conditions

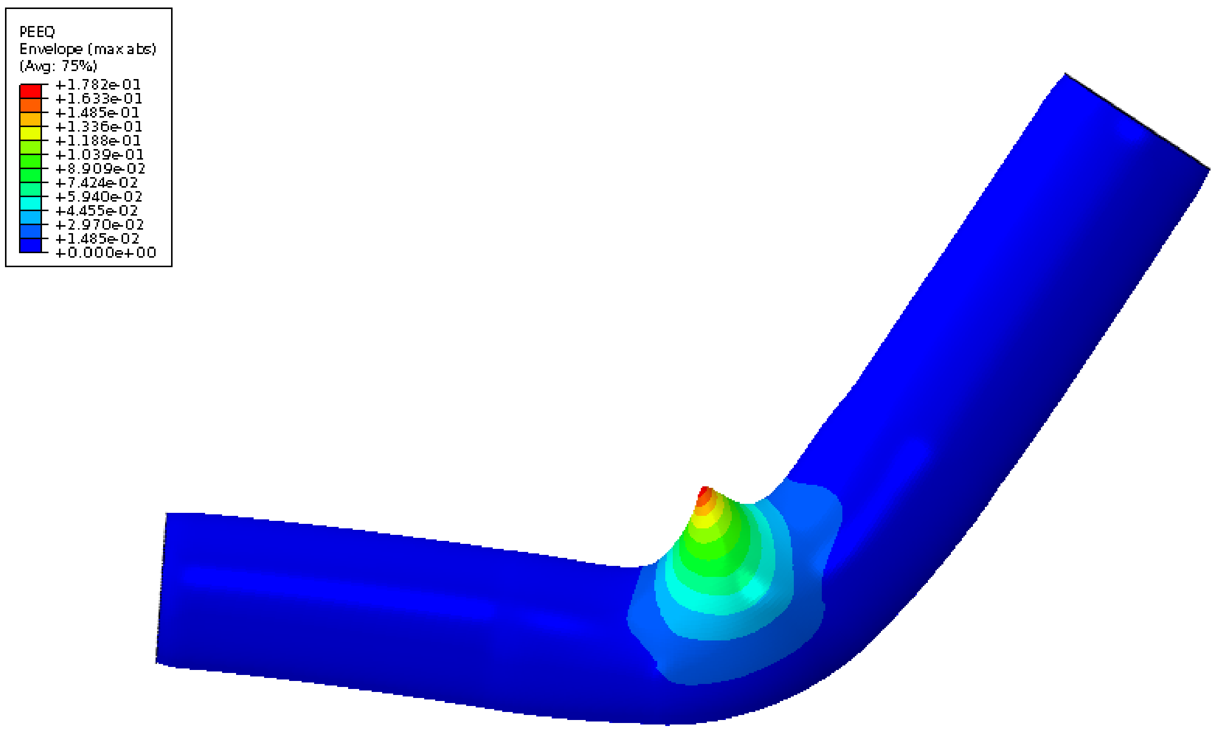

Figure 5: Plastic Strain at Failure (Deformations Magnified 20x)

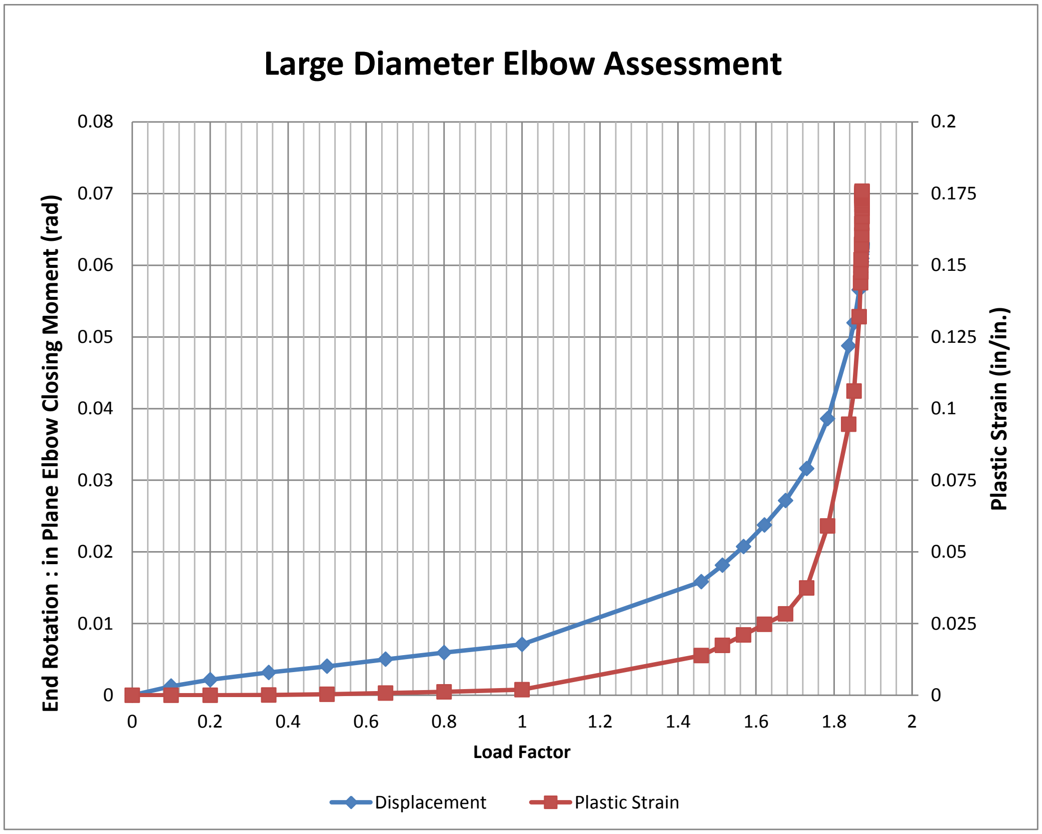

Figure 6: Plot of Load versus Displacement and Peak Plastic Strain

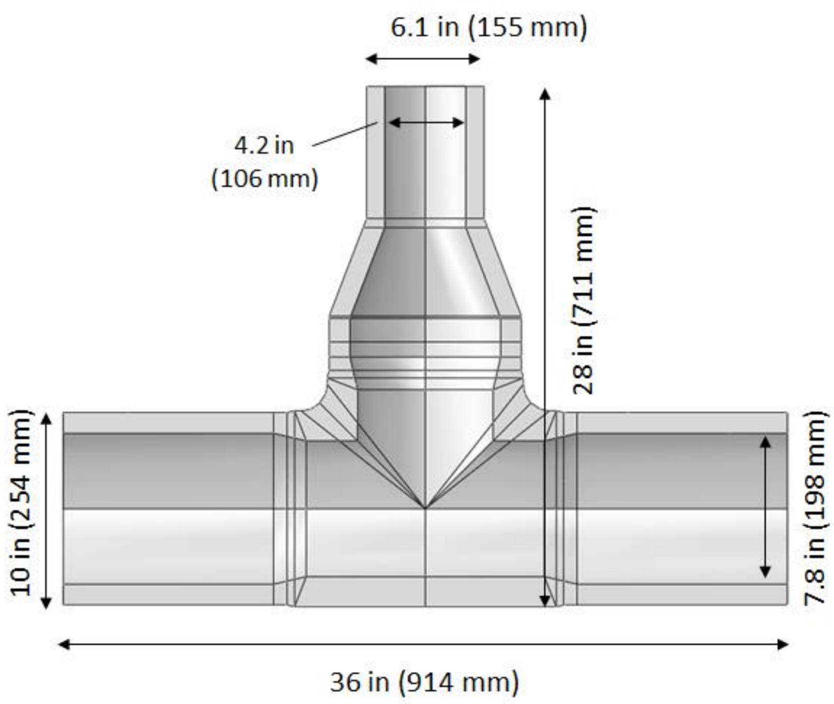

Figure 7: Tee & Reducer Nominal Dimensions

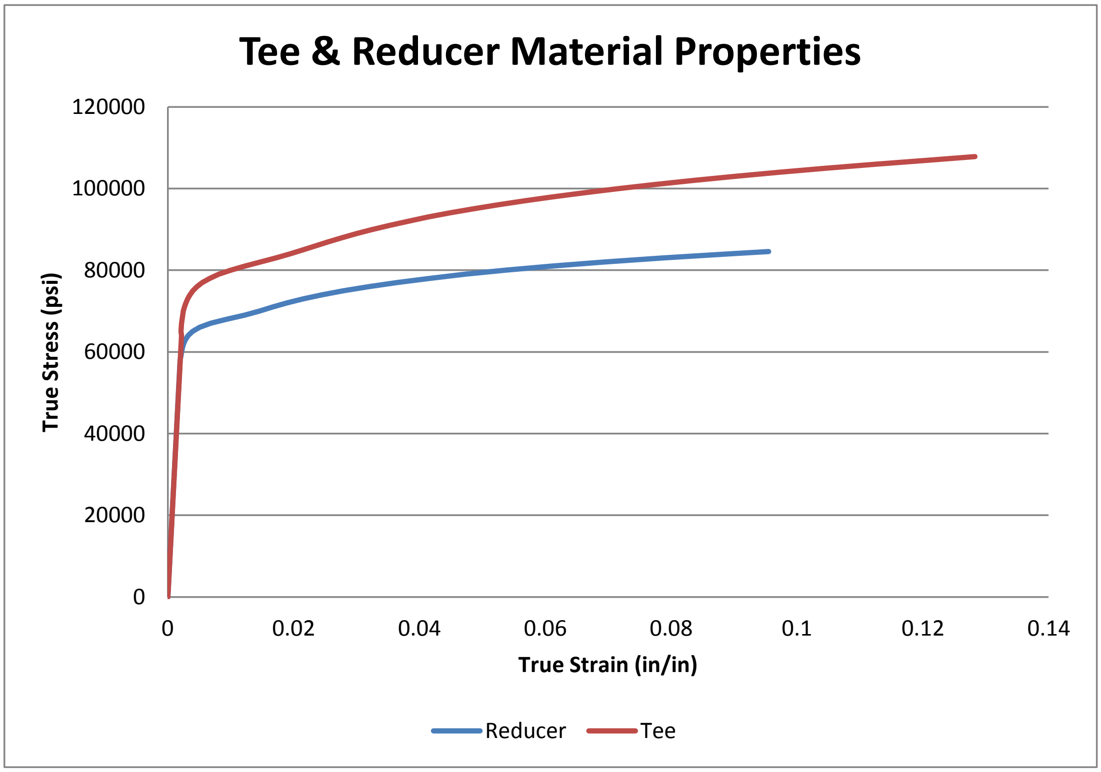

Figure 8: Tee & Reducer Material Properties

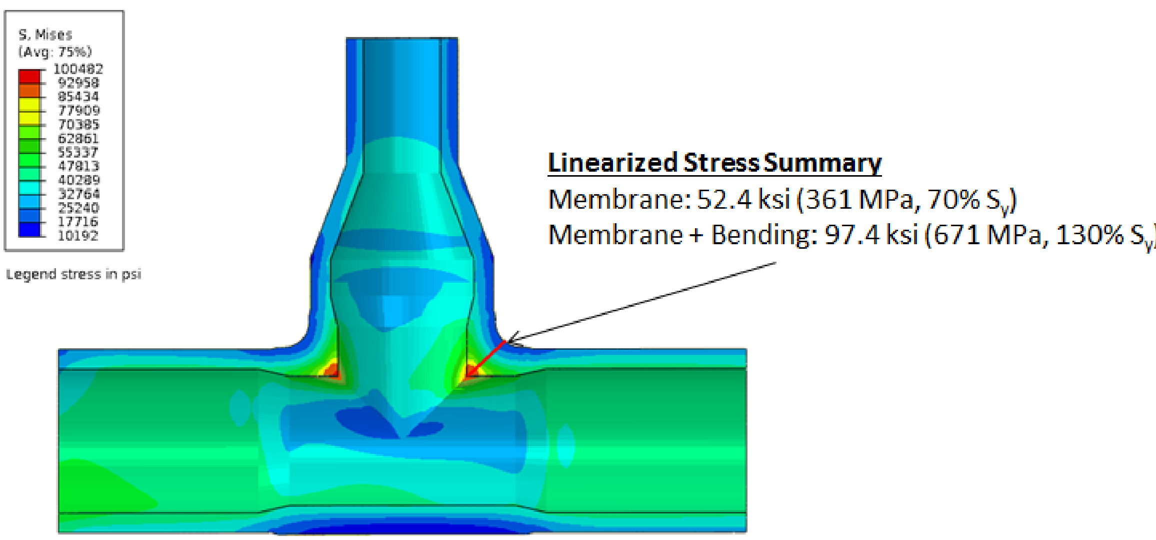

Figure 9: Tee & Reducer Stresses from Linear Elastic Analysis

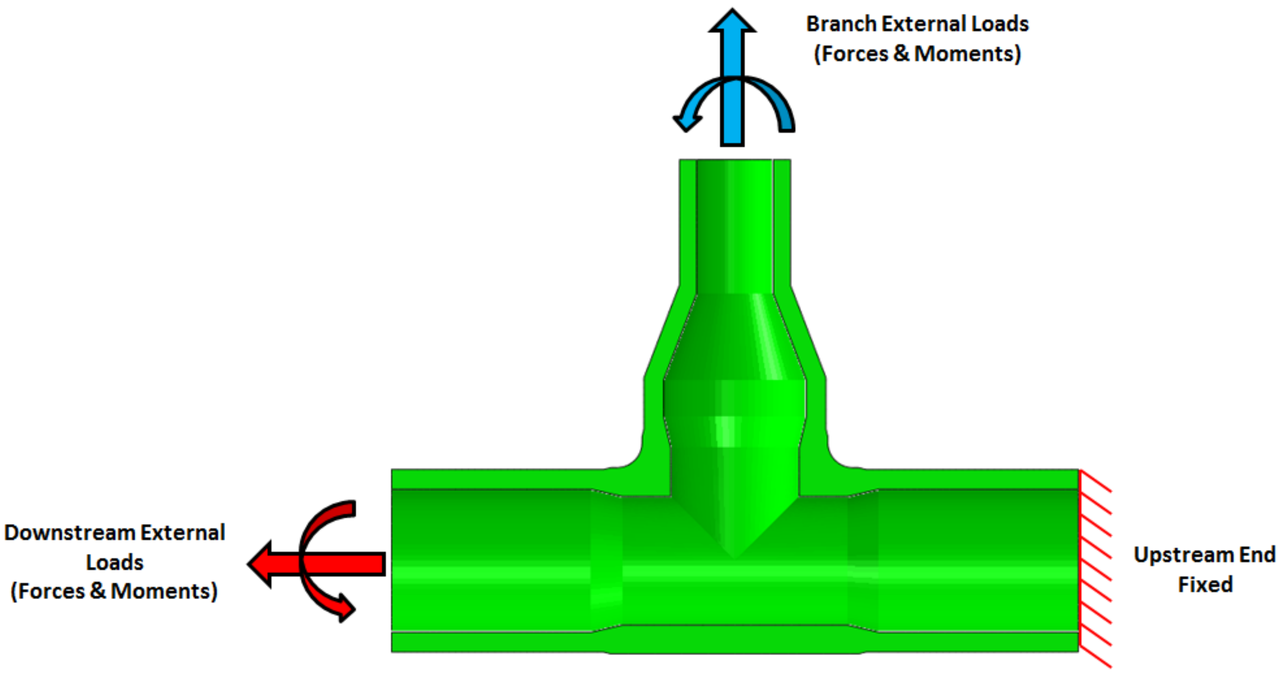

Figure 10: Tee & Reducer Applied External Loads

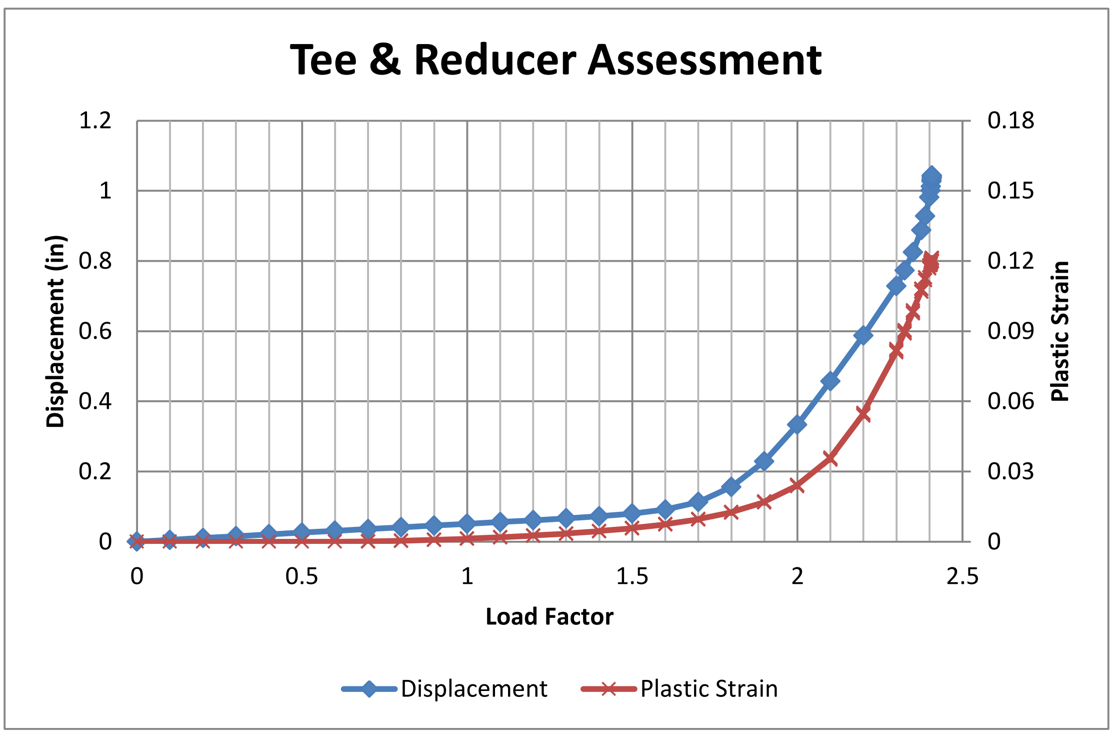

Figure 11: Plot of Load versus Displacement for Tee & Reducer

Figure 12: Full-Scale Burst Test Sample

Figure 13: Full-Scale Burst after Failure

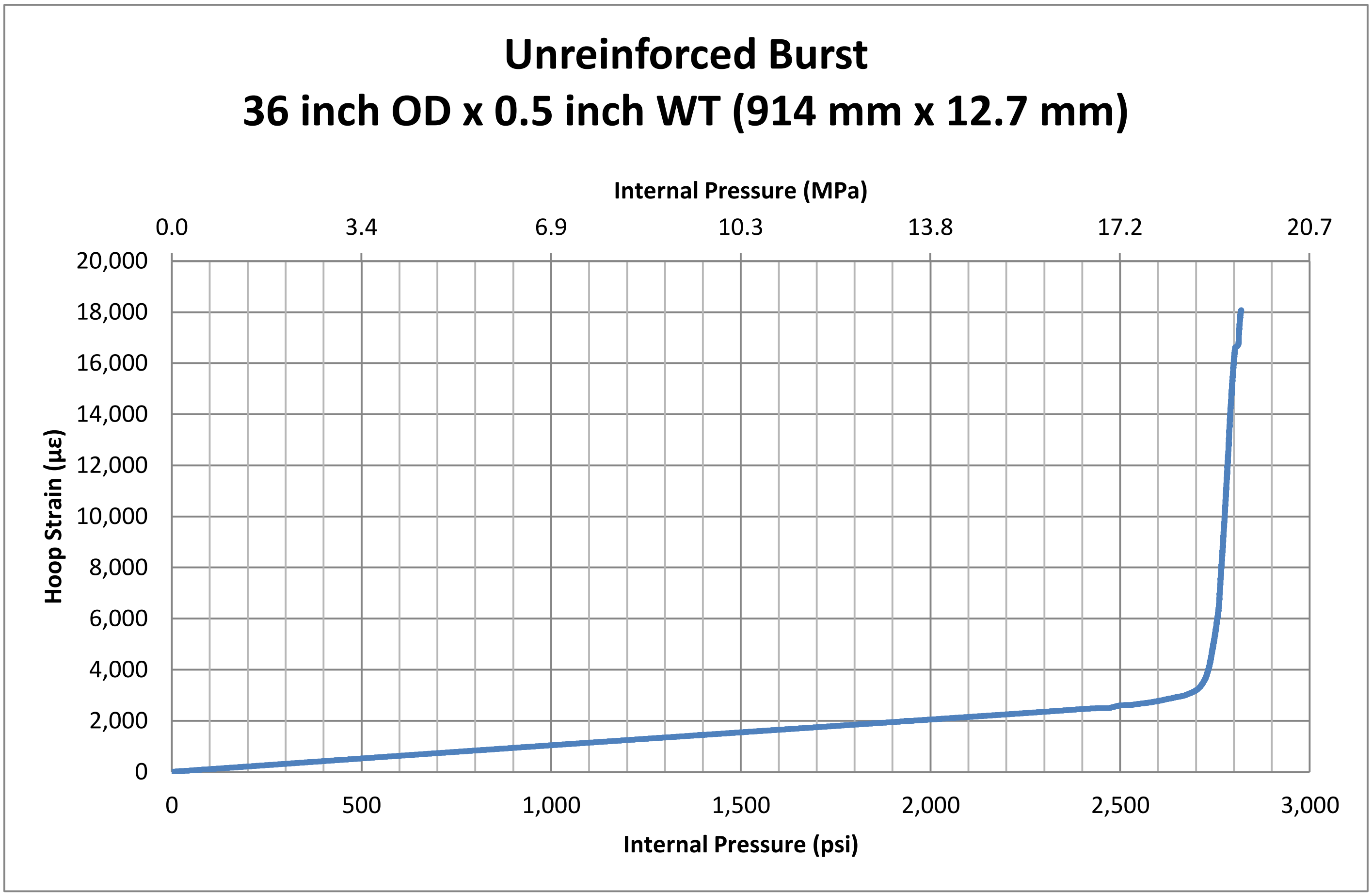

Figure 14: Burst Test Results